LCA Methodology

# Visual Data Analysis and Decision Support Methods for Non-Deterministic LCA

**Jean-Francois Le Téno**

Centre Scientifique et Technique du Bâtiment, 24, rue J. Fourier, F-38400 St Martin d'Hères, France

**Corresponding address:** Dr. Jean-Francois Le Téno; e-mail: leteno@cstb.fr

## Abstract

Principal components analysis coupled with non-parametric bootstrapping is introduced with an example as a powerful tool to help visualise, analyse and decide on comparative or single non-deterministic LCI/LCIA results. Decision support is provided by adding non-parametric bootstrapping (NPB) to the GAIA plane, which is a special case of principle comments analysis (PCA) built around the Promethée multicriteria decision aid model.
In addition to an easy to analyse visual presentation of otherwise complex and cluttered numerical results, PCA with NPB is equally at ease will all data uncertainty formats used to date in LCA.

**Keywords:** Bootstrapping; LCA; Life Cycle Assessment (LCA); multi criteria decision aid (MCDA); non-deterministic LCA; NPB; non-parametric bootstrapping (NPB); PCA; principal components analysis (PCA); Promethée; MCDA; visual data analysis

## 1 Introduction

The analysis of LCA results can be made difficult in practical situations by the quantity of data, the multiple units types, the various media to which substances outflow, judgmental values to be applied, etc. Analysis is also made difficult by the uncertainty on background and foreground data. As a direct consequence of this, LCA results can be difficult to communicate to non specialists.

In the "classical" LCA model this situation is handled by not taking uncertainty into account (possibly getting some insight afterwards through sensitivity analysis, for instance) and by producing a single index for each alternative. Doing so, comparing alternative multidimensional data sets produced by alternative LCI and LCIA becomes straightforward. This position, however, is now evolving in many ways. Not only is blind confidence in the "classical" LCI and LCIA models and their results decreasing, but also uncertainty in the input data is more often reckoned with. Thus LCA is steadily evolving into a non deterministic model.

Depending on the practitioner, non-deterministic LCAs will not always be performed up to the index calculation step. If they are, however, results are no longer multidimensional but only uncertain. Common statistical techniques can be used to document the index uncertainty and assess the relevance of the comparison of two indexes (STEEN). If an index is not calculated, the practitioner is faced with multidimensional *and* uncertain impact indicators or aggregated flows sets, thereby missing ways to get a synthetic view apart from a tedious analysis of tables like **Table 1**.

Extracting synthetic information from multidimensional (or "multivariate") data sets is the purpose of a family of data analysis techniques. Some of them have already been used in LCA. Multivariate factor analysis is mentioned by (HUELE) and principal components analysis (PCA) has been used by (LE TÉNO, 1997) on deterministic LCA results. In both works mentioned, these techniques are used to visualise "the whole thing" and analyse its inner structure. Also, (MARESCHAL, 1988) presents a special case of PCA which can be used to provide direct support for multicriteria decision making.

The purpose of this paper is to present example results of viewing, analysis and decision support with non-deterministic LCA output data through the application of PCA coupled with non-parametric bootstrap (NPB) as an uncertainty analysis technique. Examples have been produced with MatLab, a commercial numerical computation package. A brief mathematical background is given in the appendix.

## 2 Viewing Multidimensional and Uncertain Data Sets with PCA

### 2.1 PCA with certain data

For illustrative purposes, we will use comparative LCI/LCIA results from (GEMIS) (→ *Table 1*[1]). Most criteria are precise values except for the last four ones which are intervals of quali-

[1] **Table 1** is a mixture of environmental and economical criteria, as will most likely be the case in the final analysis of LCA results.

Table 1: Alternatives x Criteria table resulting from multiple LCI-LCIA

| Alternatives $A_i$ \ Criteria $C_j$ | SO2 [g] | NOx [g] | Dust [g] | CO2 [g] | CO [g] | CH4 [g] | NMVOC [g] | N2O [g] | Int. Cost [Pf] | Ext. Cost [Pf] | Waste | Hazard risk | Micro Ecoloov | Used surface |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 Diesel | 0.48 | 0.47 | 0.02 | 358.70 | 0.48 | 0.10 | 0.29 | 0.00 | 10.01 | 2.26 | [-1,0] | [-2,0] | [-2,0] | [-1,0] |
| 2 Gas | 0.03 | 0.24 | 0.01 | 256.30 | 0.67 | 1.06 | 0.35 | 0.00 | 9.18 | 1.56 | [-1,0] | [-1,0] | [-1,0] | [-1,0] |
| 3 Gas burner | 0.03 | 0.18 | 0.01 | 224.90 | 0.56 | 0.91 | 0.30 | 0.00 | 10.32 | 1.36 | [-1,0] | [-1,0] | [-1,0] | [-1,0] |
| 4 Electricity (coal) | 0.51 | 0.55 | 0.07 | 992.60 | 1.09 | 4.98 | 0.55 | 0.01 | 13.20 | 6.13 | [-3,0] | [-1,0] | [-3,0] | [-3,0] |
| 5 Electricity mix | 0.42 | 0.47 | 0.06 | 802.60 | 0.90 | 4.00 | 0.45 | 0.01 | 13.20 | 4.97 | [-3,0] | [-1,0] | [-3,0] | [-3,0] |
| 6 Heat pump + Electricity | 0.35 | 0.35 | 0.02 | 391.40 | 0.51 | 1.11 | 0.28 | 0.00 | 12.86 | 2.44 | [-3,0] | [-2,0] | [-3,0] | [-3,0] |
| 7 Heat pump + gas | 0.29 | 0.56 | 0.04 | 271.30 | 5.62 | 1.15 | 3.21 | 0.00 | 8.21 | 2.14 | [-2,0] | [-1,0] | [-3,0] | [-2,0] |
| 8 Heat pump + diesel | 0.03 | 0.15 | 0.00 | 192.50 | 0.48 | 0.77 | 0.26 | 0.00 | 9.87 | 1.16 | [-2,0] | [-1,0] | [-1,0] | [-1,0] |
| 9 Combined Elec/Heat (gas) | 0.18 | 0.78 | 0.06 | 352.90 | 7.91 | 2.18 | 4.39 | 0.00 | 11.12 | 2.81 | [-2,0] | [-1,1] | [-3,1] | [-2,1] |
| 10 Combined Elec/Heat (diesel) | 0.47 | 0.63 | 0.30 | -95.30 | -0.03 | -3.61 | 0.21 | 0.01 | 5.27 | -0.39 | [-1,3] | [-2,1] | [-2,3] | [-1,3] |
| 11 Locale generation (individual housing) | -0.13 | 0.24 | 0.00 | 11.60 | 0.58 | -0.91 | 0.21 | 0.00 | 11.74 | -0.01 | [-1,3] | [-1,1] | [-1,3] | [-1,3] |
| 12 Local generation (collective housing) | -0.18 | 0.26 | -0.01 | -36.50 | 0.62 | -1.34 | 0.20 | 0.00 | 9.13 | -0.32 | [-1,3] | [-1,1] | [-1,3] | [-1,3] |
| 13 Local generation (mixed housing) | -0.20 | 0.26 | -0.01 | -57.10 | 0.61 | -1.67 | 0.19 | 0.00 | 8.02 | -0.47 | [-1,3] | [-1,1] | [-1,3] | [-1,3] |
| 14 Plant (gas) | -0.39 | 0.01 | -0.04 | -380.40 | 0.60 | -3.55 | 0.10 | -0.01 | 5.60 | -2.54 | [-1,3] | [-1,1] | [0,3] | [-1,3] |
| 15 Plant (diesel + gas) | 0.29 | 0.29 | 0.03 | 162.90 | 0.18 | 0.52 | 0.09 | 0.00 | 6.03 | 1.15 | [-3,0] | [-1,0] | [-3,0] | [-3,0] |
| 16 Diesel + Insulation | 0.24 | 0.28 | 0.03 | 195.80 | 0.26 | 0.07 | 0.16 | 0.00 | 9.60 | 1.24 | [-1,0] | [-2,0] | [-2,0] | [-2,0] |
| 17 Bio-gas (municipal waste) | -0.18 | 0.37 | -0.01 | -477.80 | -0.02 | -2.39 | -0.21 | -0.01 | 7.76 | -2.64 | [0,2] | 0 | [-1,2] | [0,2] |
| 18 Wood | 0.15 | 0.75 | 0.06 | 50.80 | 0.90 | 0.07 | 0.29 | 0.01 | 6.63 | 0.70 | 0 | 0 | [-1,0] | 0 |
| 19 Straw | 0.21 | 0.84 | 0.28 | 58.90 | 1.04 | 0.08 | 0.25 | 0.03 | 8.11 | 0.86 | [-1,0] | 0 | [-1,0] | 0 |
| 20 Reed | 0.17 | 0.60 | 0.03 | 136.10 | 0.71 | 0.34 | 0.22 | 0.08 | 11.66 | 1.18 | [-2,0] | [-1,0] | [-2,0] | [-2,0] |

tative values. As for now, only their mean value will be considered. We will deal with their uncertainty in the next paragraph.

The purpose of PCA is to produce a planar view of the table, so that most of the information is preserved. To do so, each alternative (line) $A_i$ is considered as a point whose coordinates are given by the value on each criterion (column) (→ *Figure 1*). A plane on which the projection of the $A_i$'s will preserve most of the initial shape of the cloud of $A_i$'s is first identified. Then, correlation coefficients between criteria are projected as segments starting from the plane's origin. The computed PCA plane for **Table 1** is shown on **Figure 2**.

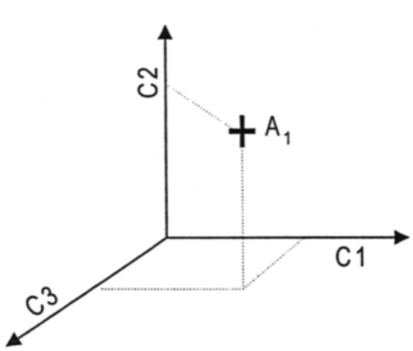


Fig. 1: Alternatives $A_i$ (matrix lines) as a point in the criteria' space (matrix columns)

Alternatives are drawn as stars and criteria as segments starting from the origin (0,0). Horizontal and vertical scales are not relevant.

Significance of the vertical and horizontal axis is in terms of synthetic criteria driving the comparison. It can be derived from the length of the criteria's projections on each axis. Thus, alternatives are ranked according to their *direct* con-

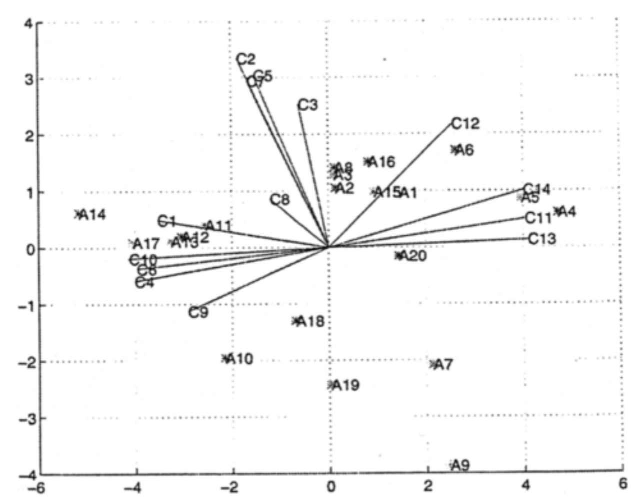


Fig. 2: PCA plane of mean of **Table 1**

tribution to the *greenhouse effect* on the horizontal axis (criteria $C_1$, $C_4$ and $C_6$) and the more we move left, the better the alternatives perform. There appears to be a cluster of rather similar alternatives in that direction ($A_{11}$, $A_{12}$, $A_{13}$, $A_{14}$, $A_{17}$). Logically enough, alternatives $A_4$ and $A_5$ (thermal electricity) are positioned at the extreme right of this axis. At the same time, the more right, the better the alternatives perform on criteria $C_{11}$, $C_{13}$ and $C_{14}$, related to *soil pollution and use*. Obviously trade-offs will have to be found between the two groups of criteria, most probably among alternatives belonging to cluster $A_1$, $A_2$, $A_3$, $A_8$, $A_{15}$ and $A_{16}$, which also perform moderately well on *other gas emissions* ($C_2$, $C_3$, $C_5$, $C_7$), or alternatively to the $A_{10}$, $A_{18}$, $A_{19}$, $A_7$ and $A_9$ group, if these gas emissions are not that important to the decider. Also, we find that $C_8$, as a short segment, is not a very discriminating criterion. This can be verified in **Table 1**, where $C_8$ is not very different between alternatives. Concerning *costs* ($C_9$ and $C_{10}$), they appear to be correlated to greenhouse gas emissions. Since we also know that $C_{10}$ (external cost) is calculated from $CO_2$ emissions in (GEMIS), this should be no surprise. Hence, $C_{10}$ is somehow redundant with $C_4$ or $C_6$. Internal costs ($C_9$) are correlated less frankly. They are, however, directly opposed to hazards ($C_{12}$). Again, some level of compromise will have to be found in a final analysis.

We see that a rather straightforward but nevertheless deep understanding of the initial data set (→ *Table 1*) can be gained from simple geometrical considerations. The same kind of analysis can be performed on a single LCI/LCIA, for instance with processes as alternatives and flows as criteria. This is partly what is done in (HUELE), as well. For a detailed description of PCA and the foundations to PCA result interpretation, readers are referred to (MURTAGH), which also provides C source code or (LEBART).

Now, how will our conclusions hold if data uncertainty of **Table 1** is taken into account ?

### 2.2 PCA with uncertain data

In case data are uncertain $A_i$'s are no longer single points but families of points contained in (hyper)cubes in the criteria coordinates system. This is illustrated in **Figure 3**. Probability distributions, fuzzy numbers or intervals apply along the (hyper)cube's edges and indicate the known distributions of values. Crisp numbers also apply. To deal with such data, PCA may produce zones of confidence around each projection of $A_i$ using either analytical techniques or simulation.

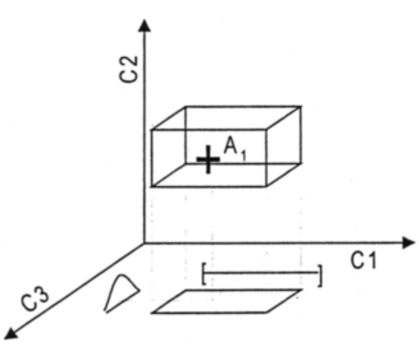


**Fig. 3:** Alternative $A_1$ with interval data on all three criteria

A very common approach is to proceed by simulation using a re-sampling technique called non-parametric bootstrapping (NPB) (LEBART). With NPB, a deterministic PCA plane is first computed using mean values. Then, values from within the (hyper)cube around each alternative are sampled according to their distributions and projected on the PCA plane, producing "clouds", the *hulls* of which indicate how variable alternatives positions are.

To proceed, **Table 1** is turned into **Table 2**. When data are intervals (last four criteria), only lower and upper bounds are sampled with a 0.5 probability each, as no information are known on values within the bounds. The first 10 criteria in **Table 1** are precise values. What we do for the purpose of this article is to *add* uncertainty according to a normal distribution centred on the mean value and extending +/-10% on each side. Adding uncertainty is a *parametric* bootstrapping that should not be used when a non deterministic LCA is performed since empirical distributions or intervals on the data are known and can be used instead of more or less arbitrary ones. 200 additional matrix are calculated and plotted on the initial PCA plane computed on mean data. Approximately 30 additional matrix is a minimum (LEBART).

PCA of **Table 1** is shown in **Figure 4**. As can be seen, alternatives are more or less spread over the plane – $A_{18}$ and $A_{19}$ being very condensed and $A_{10}$ very widespread. As of the clusters of alternatives identified in the deterministic PCA, $A_{11}$, $A_{12}$ and $A_{13}$ are too overlapped to be considered as significantly different; $A_{17}$ is a little bit more separate and much more condensed. As for $A_{14}$, it can be considered as separate from the other alternatives in the cluster.

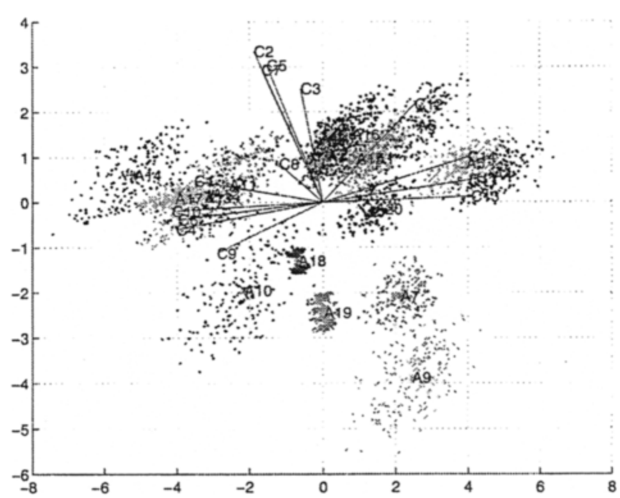


**Fig. 4:** PCA plane of **Table 2** with bootstrap

Table 2: Alternatives x Criteria table from re-sampled Table 1

| Criteria / Alternatives $A_i$ | SO2 | | NOx | | Dust | | CO2 | | CO | | CH4 | | NMVOC | | N2O | | Int. Cost [Pf] | | Ext. Cost [Pf] | | Waste | Hazard risk | Micro Ecology | Used surface |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Min | Max | Min | Max | Min | Max | Min | Max | Min | Max | Min | Max | Min | Max | Min | Max | Min | Max | Min | Max | Min, max | Min, max | Min, max | Min, max |
| 1 Diesel | 0.45 | 0.55 | 0.43 | 0.54 | 0.02 | 0.02 | 332.50 | 397.82 | 0.44 | 0.53 | 0.09 | 0.11 | 0.26 | 0.32 | 0.00 | 0.00 | 9.48 | 11.34 | 2.04 | 2.59 | [-1,0] | [-2,0] | [-2,0] | [-1,0] |
| 2 Gas | 0.03 | 0.03 | 0.22 | 0.27 | 0.01 | 0.01 | 236.29 | 289.76 | 0.61 | 0.74 | 1.00 | 1.21 | 0.32 | 0.39 | 0.00 | 0.00 | 8.59 | 10.25 | 1.46 | 1.79 | [-1,0] | [-1,0] | [-1,0] | [-1,0] |
| 3 Gas burner | 0.03 | 0.03 | 0.17 | 0.21 | 0.01 | 0.01 | 206.58 | 249.00 | 0.51 | 0.63 | 0.84 | 1.01 | 0.28 | 0.33 | 0.00 | 0.00 | 9.51 | 11.43 | 1.26 | 1.55 | [-1,0] | [-1,0] | [-1,0] | [-1,0] |
| 4 Electricty (coal) | 0.47 | 0.57 | 0.51 | 0.61 | 0.06 | 0.08 | 912.02 | 1127.25 | 1.02 | 1.23 | 4.55 | 5.62 | 0.51 | 0.62 | 0.01 | 0.01 | 12.11 | 14.66 | 5.75 | 6.91 | [-3,0] | [-1,0] | [-3,0] | [-3,0] |
| 5 Electricty mix | 0.39 | 0.48 | 0.45 | 0.52 | 0.06 | 0.07 | 746.63 | 911.36 | 0.84 | 1.00 | 3.67 | 4.53 | 0.43 | 0.51 | 0.01 | 0.01 | 11.93 | 15.07 | 4.64 | 5.48 | [-3,0] | [-1,0] | [-3,0] | [-3,0] |
| 6 Heat pump + electricity | 0.32 | 0.40 | 0.33 | 0.39 | 0.02 | 0.02 | 358.43 | 448.11 | 0.47 | 0.56 | 1.03 | 1.24 | 0.26 | 0.32 | 0.00 | 0.00 | 12.04 | 14.33 | 2.20 | 2.69 | [-3,0] | [-2,0] | [-3,0] | [-3,0] |
| 7 Heat pump + gas | 0.27 | 0.33 | 0.52 | 0.63 | 0.04 | 0.04 | 245.61 | 301.04 | 5.23 | 6.38 | 1.05 | 1.29 | 2.93 | 3.68 | 0.00 | 0.00 | 7.76 | 9.22 | 1.99 | 2.40 | [-2,0] | [-1,0] | [-3,0] | [-2,0] |
| 8 Heat pump + Diesel | 0.03 | 0.03 | 0.14 | 0.17 | 0.00 | 0.00 | 174.10 | 216.02 | 0.44 | 0.53 | 0.70 | 0.88 | 0.24 | 0.30 | 0.00 | 0.00 | 8.89 | 11.29 | 1.07 | 1.28 | [-2,0] | [-1,0] | [-1,0] | [-1,0] |
| 9 Combines Elec/heat (gas) | 0.17 | 0.20 | 0.74 | 0.86 | 0.05 | 0.07 | 327.63 | 398.48 | 7.42 | 9.09 | 1.98 | 2.46 | 4.13 | 4.91 | 0.00 | 0.00 | 10.25 | 12.63 | 2.64 | 3.21 | [-2,0] | [-1,1] | [-3,1] | [-2,1] |
| 10 Combined Elec/Heat (diesel) | 0.44 | 0.52 | 0.60 | 0.70 | 0.28 | 0.34 | -88.05 | -108.12 | -0.03 | -0.03 | -3.28 | -3.98 | 0.19 | 0.24 | 0.01 | 0.01 | 4.88 | 5.91 | -0.36 | -0.43 | [-1,3] | [-2,1] | [-2,3] | [-1,3] |
| 11 Local generation (individual housing) | -0.12 | -0.15 | 0.22 | 0.27 | 0.00 | 0.00 | 10.91 | 13.18 | 0.54 | 0.66 | -0.84 | -1.03 | 0.20 | 0.24 | 0.00 | 0.00 | 10.76 | 13.04 | -0.01 | -0.01 | [-1,3] | [-1,1] | [-1,3] | [-1,3] |
| 12 Local generation (collective housing) | -0.17 | -0.21 | 0.23 | 0.30 | -0.01 | -0.01 | -34.56 | -40.73 | 0.57 | 0.71 | -1.27 | -1.50 | 0.19 | 0.22 | 0.00 | 0.00 | 8.33 | 10.10 | -0.29 | -0.37 | [-1,3] | [-1,1] | [-1,3] | [-1,3] |
| 13 Local generation (mixed housing) | -0.18 | -0.23 | 0.24 | 0.29 | -0.01 | -0.01 | -52.96 | -63.05 | 0.57 | 0.68 | -1.54 | -1.85 | 0.17 | 0.22 | 0.00 | 0.00 | 7.32 | 8.98 | -0.45 | -0.53 | [-1,3] | [-1,1] | [-1,3] | [-1,3] |
| 14 Plant (gas) | -0.37 | -0.44 | 0.01 | 0.01 | -0.04 | -0.04 | -347.20 | -421.26 | 0.55 | 0.68 | -3.21 | -4.06 | 0.09 | 0.11 | -0.01 | -0.01 | 5.19 | 6.34 | -2.31 | -2.82 | [-1,3] | [-1,1] | [0,3] | [-1,3] |
| 15 Plant (coal + fuel) | 0.27 | 0.32 | 0.27 | 0.33 | 0.03 | 0.03 | 146.82 | 184.84 | 0.17 | 0.20 | 0.49 | 0.58 | 0.08 | 0.10 | 0.00 | 0.00 | 5.53 | 6.65 | 1.04 | 1.27 | [-3,0] | [-1,0] | [-3,0] | [-3,0] |
| 16 Diesel + insulation | 0.23 | 0.26 | 0.26 | 0.31 | 0.03 | 0.03 | 184.27 | 217.16 | 0.24 | 0.29 | 0.06 | 0.08 | 0.15 | 0.18 | 0.00 | 0.00 | 8.78 | 10.66 | 1.14 | 1.39 | [-1,0] | [-2,0] | [-2,0] | [-2,0] |
| 17 Bio-gas (municipal waste) | -0.17 | -0.20 | 0.34 | 0.41 | -0.01 | -0.01 | -431.44 | -538.23 | -0.02 | -0.02 | -2.25 | -2.67 | -0.19 | -0.23 | -0.01 | -0.01 | 7.23 | 8.71 | -2.46 | -3.00 | [0,2] | 0 | [-1,2] | [0,2] |
| 18 Wood | 0.14 | 0.17 | 0.68 | 0.83 | 0.06 | 0.07 | 48.24 | 57.23 | 0.81 | 1.03 | 0.07 | 0.08 | 0.27 | 0.32 | 0.01 | 0.01 | 6.27 | 7.43 | 0.64 | 0.79 | 0 | 0 | [-1,0] | 0 |
| 19 Straw | 0.20 | 0.24 | 0.79 | 0.94 | 0.26 | 0.32 | 53.25 | 65.59 | 0.96 | 1.18 | 0.08 | 0.09 | 0.23 | 0.28 | 0.03 | 0.03 | 7.50 | 8.93 | 0.81 | 0.96 | [-1,0] | 0 | [-1,0] | 0 |
| 20 Reed | 0.15 | 0.19 | 0.54 | 0.66 | 0.03 | 0.03 | 125.66 | 152.06 | 0.67 | 0.79 | 0.32 | 0.39 | 0.21 | 0.25 | 0.07 | 0.09 | 10.83 | 13.34 | 1.10 | 1.33 | [-2,0] | [-1,0] | [-2,0] | [-2,0] |

## 3 Supporting Decision Making through PCA

As illustrated on the examples above, a usual PCA visually provides relevant information to assist in the decision making process:

- Synthetic criteria can be identified through projection of the criteria segments on the vertical and horizontal axis, thus giving a better understanding of the global factors at play
- Highly correlated criteria can be identified and possibly simplified into fewer criteria
- Highly influent criteria and negligible ones can be identified through their length on the PCA plane
- Clusters of similar actions can be located and possibly eliminated from the analysis

Additionally, using the non parametric bootstrapping technique, more certainty can be gained as:

- The (non)-overlapping of two alternatives' hulls shows how much sense their ranking makes; alternatives which do not separate enough can be eliminated
- All available data are visualised, not only mean values
- A largely spread alternative which may deserve investment into a more precise data collection is clearly identified

In a decision context, alternatives are plotted with a view to identifying the best performing one(s), and not only to locate them as function of their input and output flows, as previously. As discussed in the **Figure 2** comments, final decision depends on a compromise between criteria, but this compromise is not explicitly nor fully handled by PCA. This is where multicriteria decision aid (MCDA) models have a role to play.

The Promethée model (BRANS) is the one used in the EQuity model for an LCA of construction products (Le Téno, 1996) as an interpretation help tool. For each criterion, Promethée is asking the user to provide:

- A weight
- Whether a minimum or a maximum is sought
- A preference function, indicating how the preference varies as function of the difference between alternatives on the given criterion

Six types of preference functions are available, allowing levels to be set as well as linear or sloppy growth in preference. Example preference function ("type 6" in Promethée) is shown on **Figure 5**.

Promethée includes a variant of PCA called "GAIA" as a visual decision support tool. Decision relevant information is requested from the user in a clear and user-friendly manner. Prior to performing deterministic PCA as usual, the decision matrix is preconditioned in a way that integrates these preference information except for the weights (→ Appendix). In addition to classical alternative and criteria plotting, a "decision vector" is drawn pointing in the direction of the "best alternatives overall" (as function of the user's criteria weights). As far as our knowledge, except for the decision vector orientation, uncertainty in the data is not handled in GAIA. What we present here is an example generalising the use of bootstrapping to deal with uncertainty in GAIA, hence providing a more complete decision support than simple NPB PCA or deterministic GAIA.

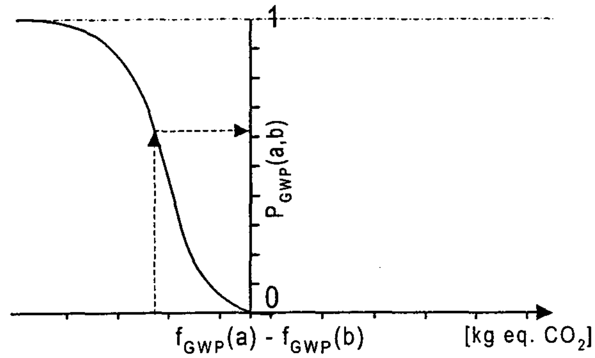


**Fig. 5:** Promethée example preference sygmoïd function ("type 6", parameter is σ) applied to the GWP indicator. a and b are alternatives.

The NPB GAIA plane generated with data from **Table 2** and preference information given in **Table 3** is shown in **Figure 6**. As can be seen with alternatives $A_{18}$ and $A_{19}$ in **Figure 4** or $A_{10}$ in **Figure 6**, clouds are sometimes made out of disconnected "stacks" of dots, and not continuous spreads. This is more a (slight) visual annoyance than a real problem. In all cases, hulls are the only relevant concept, regardless of the distributions of initial data within their bounds being continuous or not. The positions of the alternatives have changed a little from what they were in **Figure 4**, due to preference information. Given other preference information, it should always be expected that the picture will be altered to precisely reflect this information. As for the criteria, their relative "lengths" have changed since, contrary to normal PCA, preference functions vary from one to the other. In the case drawn, previous comments still hold. Supplementary information is available through the decision vector: Alternatives A2, A3, A8 are close to the best compromise direction (according the user's preferences) and could be chosen for further investigation or public debate, for instance.

## 4 Some Comments on the Modelling of Uncertainty in LCA

On a general basis, we think that resorting to crisp (mean, etc.) values in LCA/LCIA, when irrelevant, is a dangerous convenience since it may lead to unstable conclusions, hence ruining the current LCAs and LCA's predictive cred-

Table 3: Preference information on Table 2 criteria in the Promethée format

| Criteria | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Weight | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Min / max | min | min | min | min | min | min | min | min | min | min | min | min | min | min |
| Preference function type | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| Parameter ($\sigma$) | 0.36 | 0.32 | 0.12 | 466.18 | 2.56 | 2.97 | 1.46 | 0.03 | 4.38 | 2.90 | 1.36 | 0.49 | 1.38 | 1.34 |

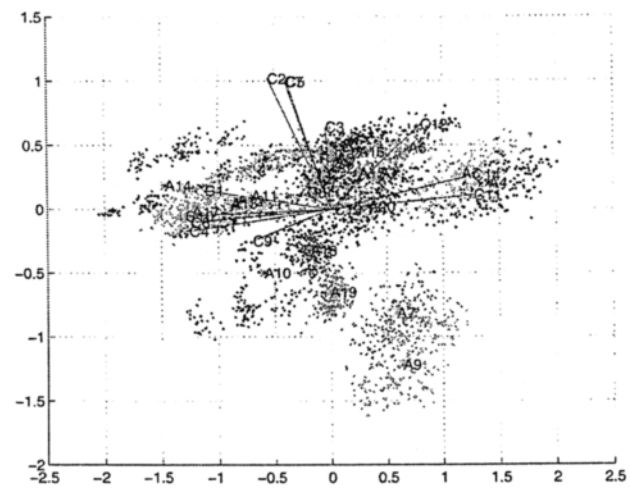


Fig. 6: GAIA plane of Table 2 with bootstrap

ibility and public reconnaissance at the same time. Hence, providing strong support for uncertainties in LCA is a priority.

As of the present situation, depending on the mathematical format chosen for uncertainty, alternative data will be intervals (LE TÉNO, 1996; HEIJUNGS), fuzzy numbers or probability distributions (COULON, STEEN). It is worth mentioning, however, that uncertainty in data is also a multidimensional concept that may have various aspects (KLIR), including:

- Fuzziness-data are vague
- Non-specificity-information are lacking
- Dissonance-conflicting information is available

Also, as shown in Table 4, all data uncertainty mathematical formats do not reckon with all uncertainty aspects. Hence, even though various non-deterministic formats are suggested for LCA, they should not be considered as mutually exclusive, but rather as complementary. Ideally, non-deterministic LCA should be able to cope with all three types of uncertainty mentioned, hence with all uncertainty formats used so far, either directly or through a common meta-format including them all.

Table 4: Uncertainty types modeled by some uncertainty models

| Uncertainty type model | Fuzziness | Non-specificity | Dissonance |
|---|---|---|---|
| Intervals | | Yes | |
| Fuzzy sets | Yes | Yes | |
| Possibility distributions | | Yes | |
| Probability distributions | | | Yes |

## 5 Conclusions

Principal components analysis (PCA) coupled with non-parametric bootstrapping (NPB) is introduced with an example as a powerful tool to help visualise, analyse and decide on comparative or single non-deterministic LCI/LCIA results. Decision support is provided by adding NPB to the GAIA plane, which is a special case of PCA built around the Promethée multicriteria decision aid model. In all cases, though PCA with NPB is equally at ease will any mixture of uncertainty formats which, aside its graphical nature, is a big advantage of this technique.

It is hoped that viewing, analysis and decision support techniques as the one presented here will foster the turning of LCA into a non-deterministic model.

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## Appendix

Mathematical foundations and necessary demonstrations are omitted for the benefit of a simpler and almost "ready to code" description of PCA and GAIA. Readers are referred to (LEBART), (MURTAGH) and (MARESCHAL, 1998) respectively for details.

**Principal components analysis**

Let R be an alternatives x criteria (n,k) matrix. Columns of R are centred around their average (centering) and reduced to unit standard deviation. Let X be the resulting matrix. Let $\lambda_{1,2}$ the first two highest eigenvalues of matrix X'X (in decreasing order) and $u_{1,2}$ their corresponding eigenvectors. ($u_1$, $u_2$) defines the first PCA plane on which projections of alternatives and criteria will be made. Coordinates ($A_i'$ and $C_j'$) of alternatives $A_i$ and criteria $C_j$ respectively in the ($u_1$,$u_2$) plane are obtained as follows:

$$A_i' = (X.u_1, X.u_2)$$

$$C_j' = \left(X'X.u_1 / \sqrt{\lambda_1}, X'X.u_2 / \sqrt{\lambda_2}\right)$$

**GAIA PCA plane**

Let R be the decision matrix. R is transformed into matrix $\Phi$ with the following equation:

$$\Phi(i,j) = \frac{1}{n-1}\sum_{b\in A}\left\{P_j(A_i,b) - P_j(b,A_i)\right\},$$

with $A_i$ = alternative number i, b an alternative belonging to A, the set of all alternatives $A_i$ and $P_j$(.,.) the preference function associated to criterion j. Then, proceed with $\Phi$ as with matrix X above. To get the decision vector, simply plot a segment from the origin to the projection of vector e defined below on the PCA plane.

$$e = \frac{1}{\sqrt{\sum_{j=1}^{k} w_j^2}}\sum_{j=1}^{k} w_j e_j$$

$w_j$ is the weight of criterion j and $e_j$ the null vector with component j set to 1.